EaD Comprehensive Lesson Plans

Strand:	Algebra	**Sub-Strand:**	Variables and Equations
Content Standard:	B7.2.3.1 Demonstrate an understanding of linear equations of the form x + a = b (where a and b are integers) by modeling problems as a linear equation and solving the problems concretely, pictorially, and symbolically.		

BASIC 7

WEEKLY LESSON PLAN – WEEK 7

<table>
<tr><td>Indicator (s)</td><td colspan="3">B7.2.3.1.3 Model linear equations, then write mathematical expressions and describe the process of solving the equation using algebraic tiles.</td><td colspan="3">Performance Indicator: Learners can solve algebraic equation.</td></tr>
<tr><td>Week Ending</td><td colspan="6">23-02-2024</td></tr>
<tr><td>Class</td><td>B.S.7</td><td>Class Size:</td><td></td><td>Duration:</td><td colspan="2"></td></tr>
<tr><td>Subject</td><td colspan="6">Mathematics</td></tr>
<tr><td>Reference</td><td colspan="6">Mathematics Curriculum, Teachers Resource Pack, Learners Resource Pack, Textbook</td></tr>
<tr><td>Teaching / Learning Resources</td><td colspan="2">Poster, Pictures, Charts, Video.</td><td colspan="2">Core Competencies:</td><td colspan="2">Ability to combine Information and ideas from several sources to reach a conclusion
Implement strategies with accuracy
Preparedness to recognize and explain results after implementation of plans</td></tr>
<tr><td>DAY/DATE</td><td>PHASE 1 : STARTER</td><td colspan="4">PHASE 2: MAIN</td><td>PHASE 3: REFLECTION</td></tr>
<tr><td>MONDAY</td><td>Discuss the mathematical model for a linear equation with the Learners.</td><td colspan="4">Assist Learners to describe the various methods of solving linear equation.
Demonstrate on how to write an equation that models a linear relationship.
Learners brainstorm to identify the steps to write linear equations.
Mathematical Model;

Mathematical modeling is the process of using various mathematical structures – graphs, equations, diagrams, scatterplots, tree diagrams, and so forth – to represent real world situations. The model provides an abstraction that reduces a problem to its essential characteristics.

Examples of Linear Equations;

3x + 4y - 7z = 2, -2x + y - z = -6, x - 17z = 4, 4y = 0, and x + y + z = 2

Ways of Writing a Linear Equation;
point-slope form
standard form
slope-intercept form.
ways to solve systems of linear equations in two variables:
graphing.
substitution method.</td><td>Through questions and answers, conclude the lesson.

Exercise;

1. What is Mathematical Model?
1. Stat 3 ways of writing a Linear equation.</td></tr>
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<td>3. elimination method.</td>
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<td>TUESDAY</td>
<td>Demonstrate on how to solve a linear equation by graphing.</td>
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1. Assist Learners to solve examples of linear equations by graphing.
2. Discuss with Learners on how to solve systems of equations by graphing and substitution
3. Assist learners to distinguish between the substitution method and the elimination method

Example;
Solve the system by graphing: {y=12x−3x−2y=4}

$$\begin{cases} y=\frac{1}{2}x-3 \\ x-2y=4 \end{cases}$$

To graph the first equation, we will use its slope and *y*-intercept

$$y=\frac{1}{2}x-3$$

$$m=\frac{1}{2}$$

$$b=-3$$

To graph the second equation, we will use the intercepts.

$$x-2y=4$$

<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>0</td><td>-2</td></tr>
<tr><td>4</td><td>0</td></tr>
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Exercise;

1. Determine whether the ordered pair is a solution to the system: {x−y=−12 x−y=−5
   i. (−2,−1)
   ii. (−4,−3)
2. Determine whether the ordered pair is a solution to the system: {3x+y=0x +2y=−5
   i. (1,−3)
   ii. (0,0)
3. Solve the system by graphing: {2x+y=7 x−2y=6

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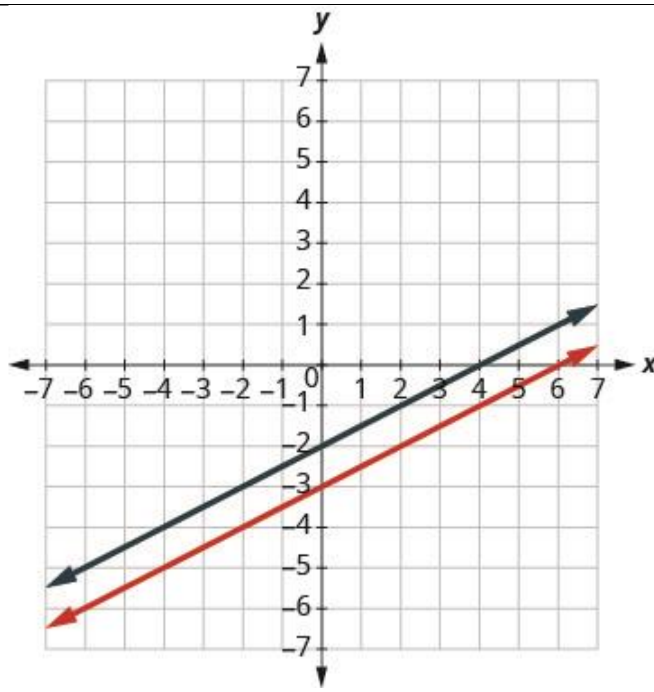


Solve a System of Equations by Elimination

The Elimination Method is based on the Addition Property of Equality. The Addition Property of Equality says that when you add the same quantity to both sides of an equation, you still have equality. We will extend the Addition Property of Equality to say that when you add equal quantities to both sides of an equation, the results are equal.

For any expressions *a*, *b*, *c*, and *d*,
To solve a system of equations by elimination, we start with both equations in standard form. Then we decide which variable will be easiest to eliminate. How do we decide? We want to have the coefficients of one variable be opposites, so that we can add the equations together and eliminate that variable.

Notice how that works when we add these two equations together:

The *y*'s add to zero and we have one equation with one variable.
Let's try another one:
This time we don't see a variable that can be immediately eliminated if we add the equations.

But if we multiply the first equation by −2, we will make the coefficients of *x* opposites. We must multiply every term on both sides of the equation by −2.

		$$\begin{cases} -2(x+4y) = -2(2) \\ 2x+5y=-2 \end{cases}$$ $$\begin{cases} -2x-8y=-4 \\ 2x+5y=-2 \end{cases}$$ Now we see that the coefficients of the *x* terms are opposites, so *x* will be eliminated when we add these two equations. Add the equations yourself—the result should be $-3y = -6$. And that looks easy to solve, doesn't it? Here is what it would look like. $$\begin{array}{r} -2x-8y=-4 \\ 2x+5y=-2 \\ \hline -3y=-6 \end{array}$$ We'll do one more: It doesn't appear that we can get the coefficients of one variable to be opposites by multiplying one of the equations by a constant, unless we use fractions. So instead, we'll have to multiply both equations by a constant. We can make the coefficients of *x* be opposites if we multiply the first equation by 3 and the second by −4, so we get 12*x* and −12*x*. $$3(4x-3y)=3(10)$$ $$-4(3x+5y)=-4(-7)$$ This gives us these two new equations: When we add these equations, the *x*'s are eliminated and we just have $-29y = 58$.	

Once we get an equation with just one variable, we solve it. Then we substitute that value into one of the original equations to solve for the remaining variable. And, as always, we check our answer to make sure it is a solution to both of the original equations.

Now we'll see how to use elimination to solve the same system of equations we solved by graphing and by substitution.

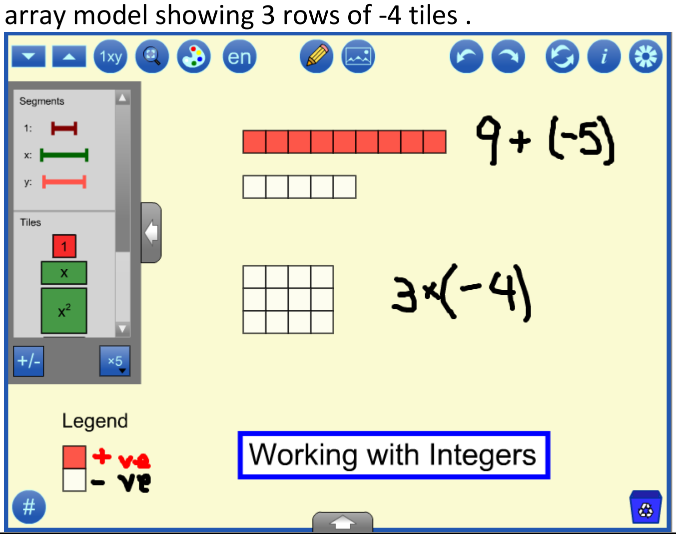
EXAMPLE

How to Solve a System of Equations by Elimination

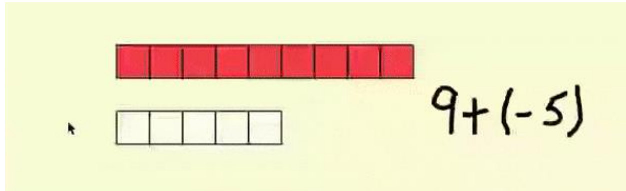
Solve the system by elimination.

Solution

Step 1. Write both equations in standard form. If any coefficients are fractions, clear them.	Both equations are in standard form, $Ax + By = C$. There are no fractions.	$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$
Step 2. Make the coefficients of one variable opposites. Decide which variable you will eliminate. Multiply one or both equations so that the coefficients of that variable are opposites.	We can eliminate the y's by multiplying the first equation by 2. Multiply both sides of $2x + y = 7$ by 2.	$\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$ $\begin{cases} 2(2x + y) = 2(7) \\ x - 2y = 6 \end{cases}$
Step 3. Add the equations resulting from Step 2 to eliminate one variable.	We add the x's, y's, and constants.	$\begin{cases} 4x + 2y = 14 \\ x - 2y = 6 \end{cases}$ $5x = 20$
Step 4. Solve for the remaining variable.	Solve for x.	$x = 4$
Step 5. Substitute the solution from Step 4 into one of the original equations. Then solve for the other variable.	Substitute $x = 4$ into the second equation, $x - 2y = 6$. Then solve for y.	$x - 2y = 6$ $4 - 2y = 6$ $-2y = 2$ $y = -1$
Step 6. Write the solution as an ordered pair.	Write it as (x, y).	$(4, -1)$

		Step 7. Check that the ordered pair is a solution to **both** original equations. — Substitute (4, −1) into $2x + y = 7$ and $x - 2y = 6$ Do they make both equations true? Yes! — $2x + y = 7$ $2(4) + (-1) \overset{?}{=} 7$ $7 = 7$ ✓ $x - 2y = 6$ $4 - 2(-1) \overset{?}{=} 6$ $6 = 6$ ✓ The solution is (4, −1).	
THURSDAY	Demonstrate on how to represent variables and constants using algebra tiles.	1. Assist Learners to represent and solve algebra problems. 2. Learners to practice substituting variables in equations and solve. 3. Discuss with the Learners on how to expand and factorize equations. **Examples** **Integers** Integers can be represented by using the 1 and -1 tiles (or segments). Arithmetical operations can be performed by manipulating two or more representations. A 1 tile can be dragged from the selection panel at the left onto the workspace. The multiplier button ×5 at the bottom right of the selection panel can be used to bring 1, 2, 5, or 10 tiles onto the workspace at a time. Existing tiles can be copied to create additional tiles in the workspace. If you select some 1 tiles, a negate button +/- appears which can be used to change them to -1 tiles. Alternatively, the negate button +/- at the bottom left of the selection panel can be used to show the opposites of tiles and segments in that panel, which can then be dragged to the workspace. In the example below, at the bottom of the workspace, the multiplication question 3 x (-4) is represented using an array model showing 3 rows of -4 tiles . 	Assist Learners to factorize algebra equations. **Exercise;** Calculate the value of x in the following equation. i.5x + 10 = 50 ii. 2x + 40 = 30 iii. 5x + 4y + 10

The addition at the top of the workspace, 9 + (-5), can be performed by combining the nine 1 tiles and the five -1 tiles. Using the tool, select the five -1 tiles and move them on top of the nine 1 tiles, the tool will “poof” the five opposites, leaving four 1 tiles. Each 1 and -1 pairing is referred to as a "zero pair".

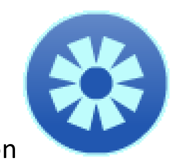


If you open and press the redo button twice  , you will see the result of “poofing” the zero pairs.
A legend is supplied to establish the colour being used for 1 and -1. These colours can be changed in the tool by:

clicking on a tile or segment in the selection panel at the left and choosing a new colour from the palette.

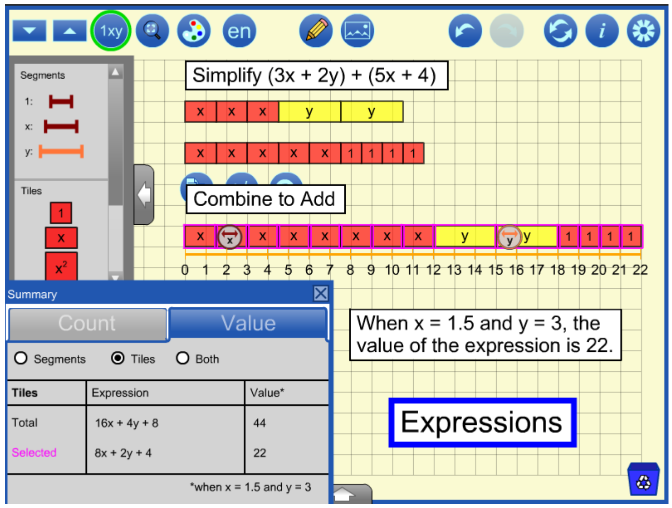
clicking the colour palette button  at the top of the screen and choosing one of six common colour sets provided.

Note:

Settings are found by clicking the Gear icon  , including a setting that:
determines if zero pairs are automatically removed.
changes the transparency of the background grid.
The values panel at the bottom of the screen can be hidden (click the arrow to show/hide it again).
The annotation tools on the right are shown by clicking the pencil icon  and can be hidden or shown using the arrow on its panel.

Expressions
Algebraic expressions can be represented using tiles or segments. Arithmetical operations can be performed by manipulating these representations. Expressions related to

area and perimeter of rectangles can be modelled directly. Adding expressions can be thought of as combining tiles. In the example below, the tiles are combined in an order that allows them to be renamed as $8x + 2y + 4$. This is a powerful visual image of what it means to 'simplify by collecting like terms'.



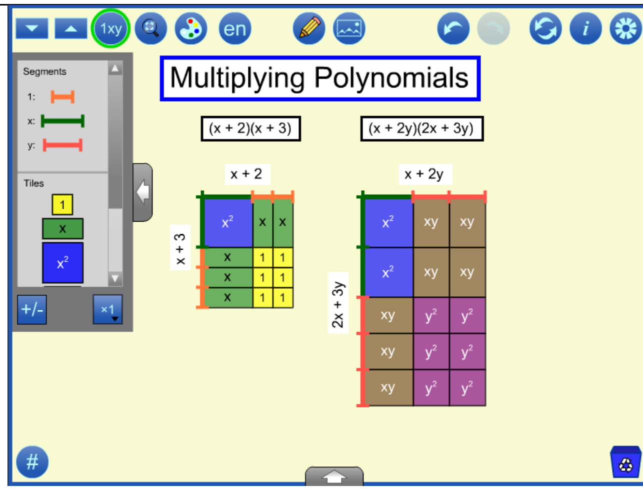
Expressions can be evaluated at specific variable values. In the example above, a number line is used to measure the length of the rectangle after *x* has been set to 1.5 and *y* to 3. Notice that each *x* tile measures 1.5 on the number line and each *y* tile measures 3 units.

The Summary dialog is shown when  is clicked. Clicking on the dialog's Value tab will allow the work to be verified. The combined tiles are selected and the value of the selected tiles is listed as 22, which agrees with the measurement. Students might be asked to explain why the value of the total is twice as big as the selected and how the expression for the total reinforces that conclusion.

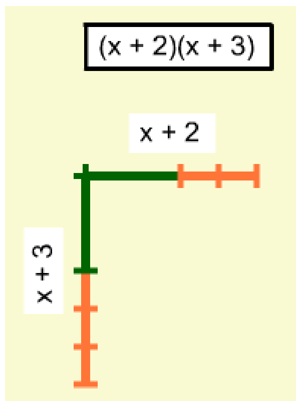
By default, opposite tiles poof (disappear) when combined, extending the "zero-pair" principle for Integers to algebraic terms. Students might be asked to represent 3x – 4 in at least six different ways.

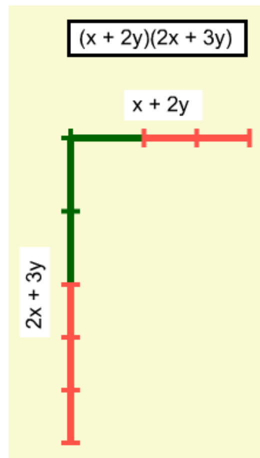
Multiplying Polynomials

Polynomials can be multiplied using an area model. A rectangle is formed with dimensions equal to the two factors. The product of the two factors is represented by the area of the rectangle and is expressed in terms of the variables in the dimensions, as in the two examples below.



Segments can be used to create the dimensions of the rectangle to be filled in.





Once the tiles are used to fill in the area, the values of the variables should be adjusted to show that the rectangular area represents the correct product for every value of the variable(s) not just specific instances. Then an expression for each product can be determined. Here the products are $x^2 + 5x + 6$ and $2x^2 + 7xy + 6y^2$. Students might be asked what is the same and different about the two multiplication questions.

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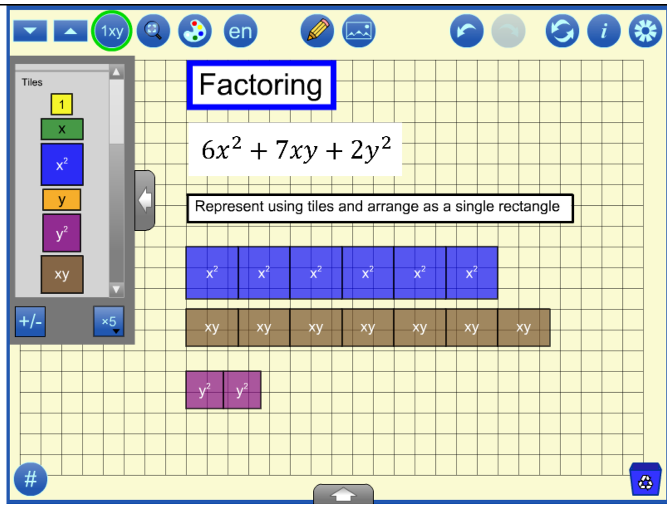
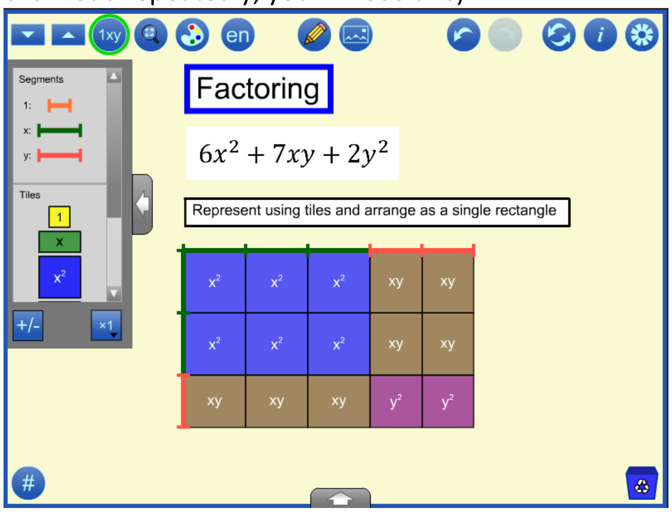
Factoring

Factoring is the inverse operation of multiplying in that the polynomial is expressed as a multiplication of two or more factors. When using an area model, this involves arranging the tiles representing the polynomial into a single rectangle. To do this with the Algebra Tiles tool, represent the polynomial then drag the tiles to form a single rectangle. Identify the dimensions of that rectangle as the factors of the polynomial.



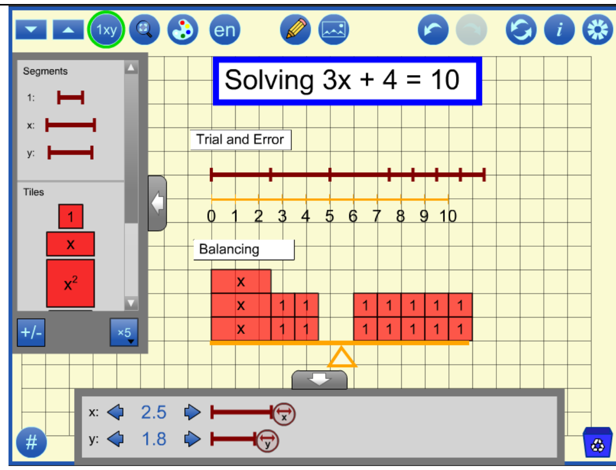
Once the rectangle is formed, segments can be used to mark off the dimensions which are used as factors in the result, (3*x* + 2*y*)(2*x* + *y*). The variable values should be adjusted to demonstrate that the factors are the same for rectangles of varying sizes. (If you open sample file 4 and click redo repeatedly, you will see this).



Solving Equations

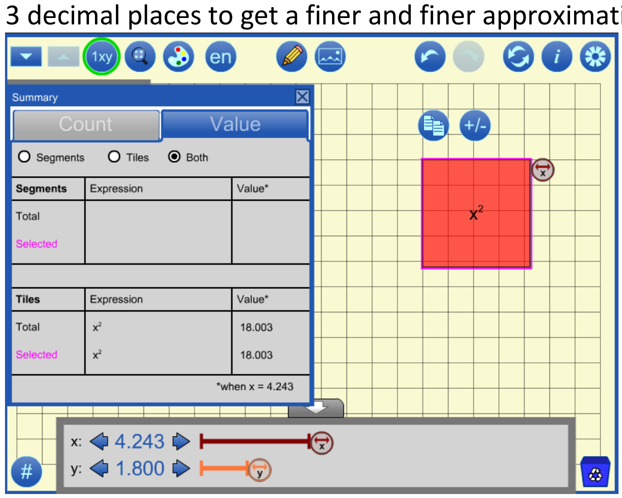
Equations can be represented using tiles or segments and solved by trial and error or by balancing.

In the trial and error example below, the equation is modeled using segments. A number line from the annotation tools is used to mark off 10 units. The value of *x* is changed by sliding the controller on the workspace or in the values panel at the bottom of the screen, until the length is 10 units. (If you open sample file 5 and click redo, you will see this).



A balance is drawn using the annotation tools, and tiles are used to represent the equation. Using the principle that equal quantities can be added or removed, 4 is removed from both sides (or -4 added and poofed). The six remaining unit tiles on the right are then arranged in 3 groups of 2 to match the left side in order to conclude that $x = 2$ (If you open [sample file 5] and continue to click redo, you will see how this might be performed using the tool).

In the example below, the equation $x^2 = 18$ is solved to find the principal square root of 18. The summary panel allows you to see the value of x^2 as x changes. Change the number of decimal places in the settings  from 1 to 2 to 3 decimal places to get a finer and finer approximation.



Trial and Error can also be used to solve a system of equations like:

$2x + y = 15$,

$x + 3y = 20$

Name of Teacher: **School:** **District:**