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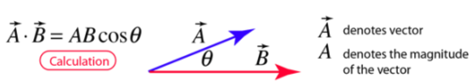
| **Strand:** | Geometry and Measurement | **Sub-Strand:** | Measurement |



**BASIC 9**

**WEEKLY LESSON PLAN – WEEK 13**

<table>
<tr><td>Content Standard:</td><td colspan="6">B9.3.2.2 Solve problems involving bearings and addition/subtraction of vectors</td></tr>
<tr><td>Indicator (s)</td><td colspan="3">B9.3.2.2.1 Show an understanding of parallel vectors and perpendicular vectors.<br>B9.3.2.2.2 Apply the triangular and parallelogram laws of addition to resolve vectors.</td><td colspan="3">Performance Indicator: Learners can find the scalar product and vector product.</td></tr>
<tr><td>Week Ending</td><td colspan="6">06-12-2024</td></tr>
<tr><td>Class</td><td>B.S.9</td><td>Class Size:</td><td></td><td>Duration:</td><td colspan="2"></td></tr>
<tr><td>Subject</td><td colspan="6">Mathematics</td></tr>
<tr><td>Reference</td><td colspan="6">Mathematics Curriculum, Teachers Resource Pack, Learners Resource Pack, Textbook.</td></tr>
<tr><td>Teaching / Learning Resources</td><td colspan="2">Poster, Cardboard, Pictures, video</td><td colspan="2">Core Competencies:</td><td colspan="2">• Communication<br>• Critical thinking<br>• Creativity and Innovation</td></tr>
<tr><td>DAY/DATE</td><td>PHASE 1 : STARTER</td><td colspan="4">PHASE 2: MAIN</td><td>PHASE 3: REFLECTION</td></tr>
<tr><td>MONDAY</td><td>Assist Learners to identify parallel and perpendicular vectors,</td><td colspan="4">1. Discuss with the Learners about how to find the scalar product of two vectors.<br>2. Learner in small groups to discuss and calculate the vector product of two vectors.<br>3. Assist Learners to find the vector forms of straight lines.<br><br>Scalar Product of Vectors<br><br>The scalar product and the vector product are the two ways of multiplying vectors which see the most application in physics and astronomy. The scalar product of two vectors can be constructed by taking the component of one vector in the direction of the other and multiplying it times the magnitude of the other vector. This can be expressed in the form:<br><br><br>If the vectors are expressed in terms of unit vectors i, j, and k along the x, y, and z directions, the scalar product can also be expressed in the form:</td><td>Learners brainstorm to find vector components given that two vectors are parallel or perpendicular.<br><br>Exercises<br><br>1. Use the formula a × b = (a2b3 − a3b2)i + (a3b1 − a1b3)j + (a1b2 − a2b1)k to find the vector product a × b in each of the following cases. (a) a = 2i + 3j, b = −2i + 9j. (b) a = 4i − 2j, b = 5i − 7j. Comment upon your solutions.<br><br>2. Use the formula in Q1 to find the vector product a × b in each of the</td></tr>
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| | | | |
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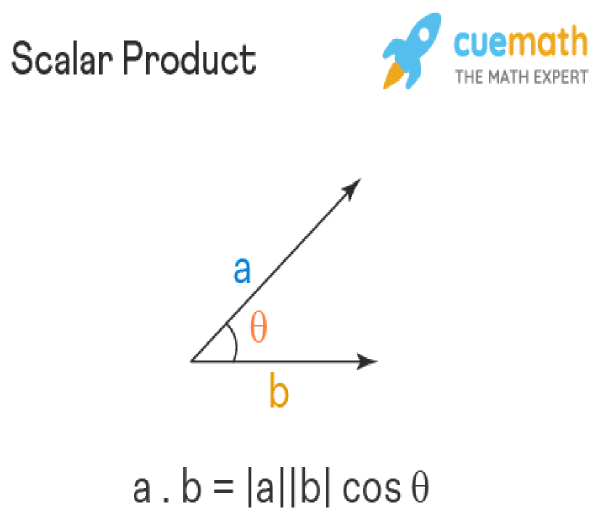
| | | $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$ where $\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$, $\vec{B} = B_x\vec{i} + B_y\vec{j} + B_z\vec{k}$<br>Applications<br><br>The scalar product is also called the "inner product" or the "dot product" in some mathematics texts.<br><br>**The Scalar Product of Two Vectors**<br><br>Consider two vectors, A and B.The angle between the two vectors is $\theta$.This is shown in the diagram below. The scalar product of A and B is equal to the magnitude of A multiplied by the magnitude of B multiplied by the cosine of the angle between them, $\theta$,which we can write asABAB⋅=\|\|\|\|($\theta$).cos<br><br>Writing two straight lines on either side of a vector symbol, for example, \|\|A,means taking the magnitude of the vector. We can write this definition in a simpler way if we just say that $A$ is the magnitude of A and $B$ is the magnitude of B:AB⋅=$AB(\theta)$<br><br> | following cases. (a) a = 5i + 3j + 4k, b = 2i − 8j + 9k. (b) a = i + j − 12k, b = 2i + j + k.<br><br>3. Use determinants to find the vector product p × q in each of the following cases. (a) p = i + 4j + 9k, q = 2i − k. (b) p = 3i + j + k, q = i − 2j − 3k.<br><br>4. For the vectors p = i + j + k, q = −i − j − k show that, in this special case, p × q = q × p.<br><br>5. For the vectors a = i + 2j + 3k, b = 2i + 3j + k, c = 7i + 2j + k, show that a × (b + c) = (a × b) + (a × c) |
| **WEDNESDAY** | Assist Learners to identify the triangle law of vector addition. | 1. Demonstrate on how to the triangle law to add two or more vectors.<br>2. Learners brainstorm to solve more examples of applying the triangle law of vector additions.<br>3. Discuss the properties of vector additions with the Learners.<br><br>**Triangle Law of Vector Addition**<br><br>A vector \\( \vec{AB} \\), in simple words, means the displacement from point A to point B. Now, imagine a scenario where a boy moves from point A to B and then from point B to C. What is the net displacement made by | Through questions and answers, conclude the lesson.<br><br>**Exercise;**<br><br>1. **What is the Commutative property** |

him from point A to C?

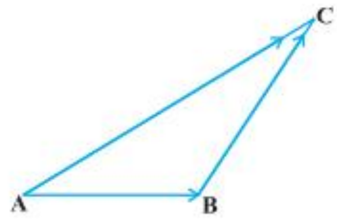



Fig. 1

This displacement is given by the vector $\vec{AC}$, where

$$\vec{AC} = \vec{AB} + \vec{BC}$$

This is the Triangle law of Vector Addition.

### Properties of Vector Addition

## Property 1 – Commutative Property

For any two vectors $\vec{a}$ and $\vec{b}$,

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

**Proof**: To prove this property, let's consider a parallelogram ABCD as shown below

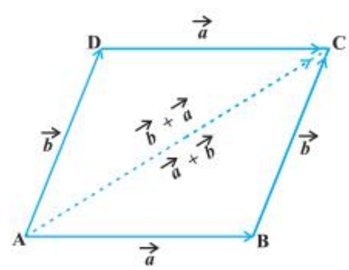



Fig. 6

Let, $\vec{AB} = \vec{a}$ and $\vec{BC} = \vec{b}$. Now, considering the triangle ABC and using the triangle law of vector addition, we have

**of vector addition?**

2. What is the Associative property of Vector addition?
3. Two vectors A and B have magnitudes of 4 units and 9 units and make an angle of 30° with each other. Find the magnitude and direction of the resultant sum vector using the triangle law of vector addition formula.
4. Two vectors with magnitudes 2 units and √2 units act on a body. The resultant vector has a magnitude of √10 units. Find the angle between the two

\( \vec{AC} \) = \( \vec{a} \) + \( \vec{b} \)

In a parallelogram, the opposite sides are always equal. Hence, we have
\( \vec{AD} \) = \( \vec{BC} \) = \( \vec{b} \) and
\( \vec{DC} \) = \( \vec{AB} \) = \( \vec{a} \)

Next, considering the triangle ADC and using the triangle law of vector addition, we have
\( \vec{AC} \) = \( \vec{AD} \) + \( \vec{DC} \) = \( \vec{b} \) + \( \vec{a} \)

Hence, \( \vec{a} \) + \( \vec{b} \) = \( \vec{b} \) + \( \vec{a} \)

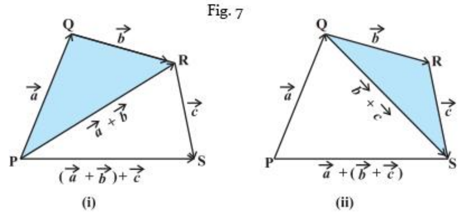
## Property 2 – Associative Property

For any three vectors \( \vec{a} \), \( \vec{b} \), and \( \vec{c} \),

(\( \vec{a} \) + \( \vec{b} \)) + \( \vec{c} \) = \( \vec{a} \) + (\( \vec{b} \) + \( \vec{c} \))

**Proof:** To prove this property, let's look at two figures as given below

Fig. 7




Let, the vectors \( \vec{a} \), \( \vec{b} \), and \( \vec{c} \) be represented by \( \vec{PQ} \), \( \vec{QR} \), and \( \vec{RS} \), respectively. Now,
\( \vec{a} \) + \( \vec{b} \) = \( \vec{PQ} \) + \( \vec{QR} \) = \( \vec{PR} \)

Also, \( \vec{b} \) + \( \vec{c} \) = \( \vec{QR} \) + \( \vec{RS} \) = \( \vec{QS} \)
Hence, (\( \vec{a} \) + \( \vec{b} \)) + \( \vec{c} \) = \( \vec{PR} \) + \( \vec{RS} \) = \( \vec{PS} \)
And, \( \vec{a} \) + (\( \vec{b} \) + \( \vec{c} \)) = \( \vec{PQ}

given vectors.

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| | | $) + $\vec{QS}$ = $\vec{PS}$<br><br>Therefore, we conclude that<br>($\vec{a}$ + $\vec{b}$) + $\vec{c}$ = $\vec{a}$ + ($\vec{b}$ + $\vec{c}$)<br><br>Note: The associative property of vector addition enables us to write the sum of three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ without using any brackets:<br>$\vec{a}$ + $\vec{b}$ + $\vec{c}$<br><br>Also, for any vector $\vec{a}$, we have<br>$\vec{a}$ + $\vec{0}$ = $\vec{0}$ + $\vec{a}$ = $\vec{a}$<br><br>The zero vector is also called the additive identity for vector addition | |
| **FRIDAY** | Discuss with the Learners about the parallelogram law of vector additions. | 1. Demonstrate on how to apply the parallelogram law of vector additions.<br>2. Assist Learners to add the vertices of parallelograms by applying the parallelogram law of vector additions.<br>3. Learners in small groups to discuss and solve more examples of adding vertices of parallelograms.<br><br>**Parallelogram Law of Vector Addition**<br><br>Now, let's consider a slightly complex scenario. Imagine a boat in a river going from one bank to the other in a direction perpendicular to the flow of the river. This boat has two velocity vectors acting on it:<br><br>1. The velocity imparted to the boat by its engine<br>2. The velocity of the flow of the river.<br><br>When these two velocities simultaneously influence the boat, it starts moving with a different velocity. Let's look at how we can calculate the resultant velocity of the boat.<br><br>To find the answer, let's take two vectors $\vec{a}$ and $\vec{b}$ shown below, as the two adjacent sides of a | Summarize the lesson.<br><br>**Exercise;**<br>1. Consider two vectors A and B. Let A have a magnitude of 4 units and point in the north direction (upwards), and let B have a magnitude of 3 units and point in the east direction (rightwards). To find the resultant vector using the parallelogram law, we construct a parallelogram with A and B as its adjacent sides. The diagonal of the parallelogram |

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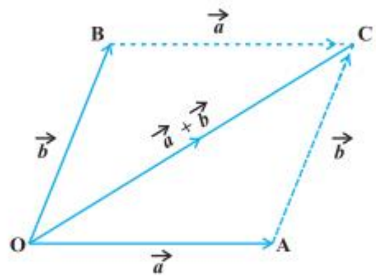
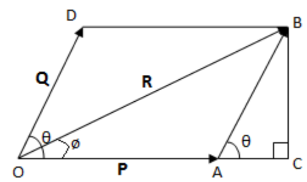
| | | parallelogram in their magnitude and direction.<br><br><br>Fig. 5<br><br>Their sum, $\vec{a}$ + $\vec{b}$ is represented in magnitude and direction by the diagonal of the parallelogram through their common point. This is the Parallelogram law of vector addition.<br><br>Note: Using the Triangle law, we can conclude the following from Fig. 5<br><br>$\vec{OA}$ + $\vec{AC}$ = $\vec{OC}$ Or,<br>$\vec{OA}$ + $\vec{OB}$ = $\vec{OC}$ … since $\vec{AC}$ = $\vec{OB}$<br><br>Hence, we can conclude that the triangle and parallelogram laws of vector addition are equivalent to each other.<br><br>Statement of Parallelogram Law<br><br>*If two vectors acting simultaneously at a point can be represented both in magnitude and direction by the adjacent sides of a parallelogram drawn from a point, then the resultant vector is represented both in magnitude and direction by the diagonal of the parallelogram passing through that point.*<br>Derivation of the law<br><br>Note: All the letters in bold represent vectors and normal letters represent magnitude only.<br><br>Let **P** and **Q** be two vectors acting simultaneously at a point and represented both in magnitude and direction by two adjacent sides OA and OD of a parallelogram OABD as shown in figure. | represents the resultant vector. By measuring the magnitude and direction of the diagonal, we determine the resultant vector.<br>2. Let's say we have vector C with a magnitude of 5 units pointing in the northeast direction (45 degrees between north and east). Similarly, we have vector D with a magnitude of 2 units pointing in the southwest direction (45 degrees between south and west). By constructing a parallelogram with C and D as its adjacent sides, the diagonal of the parallelogram represents the resultant vector. Measuring the magnitude and direction of the diagonal gives us the resultant vector. |

Let θ be the angle between **P** and **Q** and **R** be the resultant vector. Then, according to parallelogram law of vector addition, diagonal OB represents the resultant of **P** and **Q**.




So, we have

**R = P + Q**

Now, expand A to C and draw BC perpendicular to OC.

From triangle OCB,

$$OB^2 = OC^2 + BC^2$$
$$or, OB^2 = (OA + AC)^2 + BC^2 \quad ......(i)$$

In triangle ABC,

$$\cos\theta = \frac{AC}{AB}$$
$$or, AC = AB\cos\theta$$
$$or, AC = OD\cos\theta = Q\cos\theta \quad [\because AB = OD = Q]$$

Also,

$$\cos\theta = \frac{BC}{AB}$$
$$or, BC = AB\sin\theta$$
$$or, BC = OD\sin\theta = Q\sin\theta \quad [\because AB = OD = Q]$$

**Magnitude of resultant:**

Substituting value of AC and BC in (i), we get

$$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$
$$or, R^2 = P^2 + 2PQ\cos\theta + Q^2\cos^2\theta + Q^2\sin^2\theta$$
$$or, R^2 = P^2 + 2PQ\cos\theta + Q^2$$
$$\therefore R = \sqrt{P^2 + 2PQ\cos\theta + Q^2}$$

which is the magnitude of resultant.

**Direction of resultant:** Let ø be the angle made by resultant **R** with **P**. Then,

From triangle OBC,

$$\tan\phi = \frac{BC}{OC} = \frac{BC}{OA + AC}$$
$$or, \tan\phi = \frac{Q\sin\theta}{P + Q\cos\theta}$$
$$\therefore \phi = \tan^{-1}(\frac{Q\sin\theta}{P + Q\cos\theta})$$

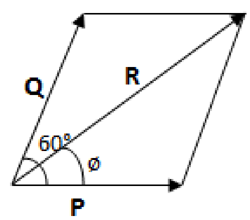
which is the direction of resultant.

Numerical Problem

Two forces of magnitude 6N and 10N are inclined at an angle of 60° with each other. Calculate the magnitude of resultant and the angle made by resultant with 6N force.

**Solution:**

Let **P** and **Q** be two forces wih magnitude 6N and 10N respectively and θ be angle between them. Let **R** be the resultant force.




So, P = 6N, Q = 10N and θ = 60°

We have,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$
$$or, R = \sqrt{6^2 + 10^2 + 2.6.10\cos 60^\circ}$$
$$\therefore R = \sqrt{196} = 14N$$

which is the required magnitude

Let ø be the angle between **P** and **R**. Then,

$$\tan\phi = \frac{Q\sin\theta}{P + Q\cos\theta}$$
$$or, \tan\phi = \frac{10\sin 60^\circ}{6 + 10\cos 60^\circ}$$
$$or, \tan\phi = \frac{5\sqrt{3}}{11}$$
$$\therefore \phi = \tan^{-1}(\frac{5\sqrt{3}}{11})$$

which is the required angle.

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**Name of Teacher:** **School:** **District:**